1. State the axioms of probability

Axioms probability [Kolmogorov's axioms]

1. $P[E] \geq 0$ for any event E
2. $P[\bar{S}] = 1$, where S is sample space
3. If E_1, E_2 are mutually exclusive events, then $P[E_1 \cup E_2 \cup \dots] = P[E_1] + P[E_2] \dots$

b] A and B alternately throw a pair of dice......
.... A winning the game

When two dices are thrown, we have

$$n(S) = 36$$

The probability of A throwing 6

$$P[A] = \frac{5}{36}$$

The probability of A not throwing 6

$$P[\bar{A}] = 1 - P[A]$$

$$= 1 - \frac{5}{36} \Rightarrow \frac{31}{36}$$

The probability of B throwing 6

$$P[B] = \cancel{\frac{6}{36}} \ \frac{1}{6}$$

The probability of B not throwing 6

$$P[\bar{B}] = 1 - P[B]$$

$$= 1 - \cancel{6}\frac{1}{6} \Rightarrow 5/6$$

The probability of A winning each

$\Rightarrow P(A) + P(\bar{A})P(\bar{B})P(A) + P(\bar{A})P(\bar{B})P(\bar{A})P(\bar{B})P(A) + \dots\dots$

$\Rightarrow \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + \frac{31}{36} \times \frac{5}{6} \times \frac{31}{36} \times \frac{5}{6} \times \frac{5}{36} + \dots$

$\Rightarrow \frac{5}{36}\left[1 + \left(\frac{31}{36} \times \frac{5}{6}\right] + \left[\frac{31}{36} \times \frac{5}{6}\right]^2 + \dots\right]$

$\therefore 1 + x + x^2 \dots\dots = \frac{1}{1-x}$

$\Rightarrow \frac{5}{36} \cdot \frac{1}{1 \cdot \left[\frac{31}{36} \times \frac{5}{6}\right]} \Rightarrow \frac{\frac{5}{36}}{1 \cdot \left[\frac{155}{216}\right]}$

$\Rightarrow \frac{\frac{5}{36}}{\frac{216-155}{216}} = \frac{\frac{5}{36}}{\frac{61}{216}}$

$\Rightarrow \frac{5}{\cancel{36}_{6}} \times \frac{\cancel{216}^{\cancel{36}^{6}}}{61}$

$\Rightarrow \frac{30}{61}$

2. What are the different types of random variable. Give two examples for each

Random variables are categorized into two primary types: Discrete and Continous

Discrete: A discrete random variable has a finite or countable infinite number of possible values

Ex: The number of heads when flipping a coin multiple times

The outcome of rolling a die

Continous: A continous random variable can take on an infinite number of value within a specific range or interval

Ex: The height of a person

The average rainfall in a region

] A random variable is distributed at random .... Find the expected value and variance of x

Given $P(X=0) = 1-P$

$P(X=1) = P$

Expected value

$E[x] = \sum x\, p(x)$

$= 0 \times P[X=0] + 1 \times P[X=1]$

$= 0 \times [1-P] + 1 \times P$

$= P$

Variance of $X = E[x^2] - [E[x]]^2$

$E[x^2] = \sum x^2 p(x)$

$= 0^2 \times [1-P] \times 1^2 \times P$

$= P$

$Var[x] = P - [P]^2$

$Var[x] = P[1-P]$

3. Let X denote the minimum of the two numbers
.... i] probability Distribution
ii] Expectation

Sol: When two dice are thrown, total no: of sample space

$\Rightarrow n(S) = 36$

$S = \{(1,1), (2,2), \ldots \ldots (6,6)\}$

If X denotes the minimum number of the two dice

The minimum number can be

1, 2, 3, 4, 5, 6

For minimum 1

(1, 1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (3,1) (4,1) (5,1) (6,1)

$P[X=1] = \frac{11}{36}$

For minimum 2

(2,2) (2,3) (2,4) (2,5) (2,6)
(3,2), (4,2) (5,2) (6,2)

$P[X=2] = \frac{9}{36}$

Similarly

$P[X=3] = \frac{7}{36}$ $\quad$ $P[X=5] = \frac{3}{36}$

$P[X=4] = \frac{5}{36}$ $\quad$ $P[X=6] = \frac{1}{36}$

The probability distribution is

X	1	2	3	4	5	6
P[x]	11/36	9/36	7/36	5/36	3/36	1/36

ii] Expectation

$E[x] = \Sigma x \, px$ ||F

$= 2.5278$

b] Given $f[x] = \begin{cases} a x^2 & 0 < x < 1 \\ 0 & \text{else where} \end{cases}$

Find constant, mean and variance of x

We know that

$\Rightarrow \int_0^1 f[x] \, dx = 1$ ||F

$\Rightarrow \int_0^1 a x^{2} = 1$

$\Rightarrow a \int_0^1 x^2 = 1$

$\Rightarrow a \left[\frac{x^3}{3} \right]_0^1 = 1$

4. A continous random variable X has probability .... $f[x] = a+bx,\ 0 \le x \le 1$
AND 0, otherwise
If the of the mean of the distribution is $1/3$. find the value of a and b

$$\Rightarrow \int_0^1 f(x)\,dx = 1$$

$$\Rightarrow \int_0^1 a+bx\,dx = 1$$

$$\Rightarrow \int_0^1 a\,dx + \int_0^1 bx\,dx = 1$$

$$\Rightarrow a\int_0^1 1\,dx + b\int_0^1 x\,dx = 1$$

$$\Rightarrow a\,[x]_0^1 + b\left[\frac{x^2}{2}\right]_0^1 = 1$$

$$\Rightarrow a\,[1-0] + b\left[\frac{1^2}{2} - \frac{0^2}{2}\right] = 1$$

$$\Rightarrow a\,[1] + b\left[\frac{1}{2}\right] = 1$$

$$\Rightarrow a + \frac{b}{2} = 1$$

$$\Rightarrow \frac{2a+b}{2} = 1$$

$$\Rightarrow 2a+b = 2 \longrightarrow (1)$$

$\therefore$ For sum = 3
(2,1) (1,2)
$P[X=3] = \frac{2}{36}$

$\therefore$ For sum = 4
(2,2) (3,1) (1,3)
$P[X=4] = \frac{3}{36}$

$\therefore$ Similarly $P[X=5] = \frac{4}{36}$
$P[X=6] = \frac{5}{36}$
$P[X=7] = \frac{6}{36}$
$P[X=8] = 5/36$
$P[X=9] = 4/36$
$P[X=10] = 3/36$
$P[X=11] = 2/36$
$P[X=12] = 1/36$

X	2	3	4	5	6	7	8	9	10	11	12
PX	1/36	2/36	3/36	4/36	5/36	6/36	5/36	4/36	3/36	2/36	1/36

Mean
$\Rightarrow E[x] = \sum x\, px$
$= 7$

Variance of x
$Var[x] = E[x^2] - [E[x]]^2$
$E[x^2] = \sum x^2 px$
$= \frac{1974}{36} \Rightarrow 54.83$

Similarly

$P[X=3] = \frac{7}{36}$ $\quad$ $P[X=5] = \frac{3}{36}$

$P[X=4] = \frac{5}{36}$ $\quad$ $P[X=6] = \frac{1}{36}$

The probability distribution is

X	1	2	3	4	5	6
$P[X]$	$^{11}/_{36}$	$^{9}/_{36}$	$^{7}/_{36}$	$^{5}/_{36}$	$^{3}/_{36}$	$^{1}/_{36}$

ii] Expectation

$E[x] = \Sigma\, x\, px$

$= 2.5278$

b] Given $f[x] = \begin{cases} a x^2 & 0 < x < 1 \\ 0 & \text{else where} \end{cases}$

Find constant, mean and variance of x

We know that

$\Rightarrow \int_0^1 f[x]\, dx = 1$

$\Rightarrow \int_0^1 a x^2 dx = 1$

$\Rightarrow a \int_0^1 x^2 = 1$

$\Rightarrow a \left[\frac{x^3}{3} \right]_0^1 = 1$

b] If X and Y are two independent r.v.s ...

such $E[2X+Y]=0$

$E[X+2Y]=33$ find $E[X]$, $E[Y]$

$E[2X+Y]=0 \longrightarrow ①$

$E[X+2Y]=33 \longrightarrow ②$

From eq ①

$E[2X+Y]=0$

$2E[X]+E[Y]=0$

$E[Y]=-2E[X] \longrightarrow ③$

From eq ②

$E[X+2Y]=33$

$E[X]+2E[Y]=33$

From eq ③

$E[X]+2[-2E[X]]=33$

$E[X]-4E[X]=33$

$-3E[X]=33$

$E[X]=-11$

Sub in eq ③

$E[Y]=-2[-11]$

$=22$

$$\Rightarrow a\left[\frac{(1)^3}{3}-\frac{(0)^3}{3}\right]=1$$

$$\Rightarrow a\left[\frac{1}{3}\right]=1$$

$$\Rightarrow a = 3$$

Mean

$$\Rightarrow \mu = \int_0^1 x\, f(x)\, dx$$

$E[X]$

$$= \int_0^1 x\, a x^2\, dx$$

$$= \int_0^1 x\, 3x^2\, dx$$

$$= 3\int_0^1 x^3\, dx$$

$$= 3\left[\frac{x^4}{4}\right]_0^1$$

$$= 3\left[\frac{(1)^4}{4}-\frac{(0)^4}{4}\right]$$

$$\mu = \frac{3}{4}$$

Variance of x

$$Var[X] = E[x^2]-[E[x]]^2$$

$$E[x^2] = \int_0^1 x^2 f[x]\, dx$$

$$= \int_0^1 x^2 a x^2\, dx$$

$$= \int_0^1 x^2 \, 3x^2 \, dx$$

$$= 3\int_0^1 x^4 \, dx$$

$$= 3\left[\frac{x^5}{5}\right]_0^1$$

$$= 3\left[\frac{(1)^5}{5} - \frac{(0)^5}{5}\right]$$

$$= \frac{3}{5}$$

$$Var[X] = \frac{3}{5} - \left[\frac{3}{4}\right]^2$$

$$= \frac{3}{5} - \frac{9}{16}$$

$$= \frac{48-45}{80} = \frac{3}{80}$$

b] If X and Y are two independent r.v. ......
.... find the variance of z, where
$Z = X - Y$

Given $Var[X] = 1$
$Var[Y] = 2$

By property of variance

$Var[X+Y] = Var[X] + Var[Y]$

$Var[Z] = Var[X-Y]$
$= Var[X] + Var[Y]$
$= 1 + 2$
$= 3$

5. Two dice are thrown, the random variable .... Write the distribution. Find the mean and variance of x

Let x be the random variable

The sum be = 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

∴ For sum = 2
(1, 1)

∴ $P[X=2] = \frac{1}{36}$